NUMERICAL HYDRODYNAMIC MODELLING OF RIVERS FOR FLOOD FORECASTING BY THE NATIONAL WEATHER SERVICE

By

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INTRODUCTION

The National Weather Service (NWS) hydrology program provides flood and daily river forecasts to the general public. Thirteen River Forecast Centers prepare the forecasts for dissemination throughout the United States.

In the late 1960's the NWS Hydrologic Research Laboratory began developing the National Weather Service River Forecast System (NWSRFS) which is based on a conceptual soil-moisture catchment model (Burnash, et al., 1973) and a snow accumulation-ablation model (Anderson, 1973). Where the runoff generated by the conceptual models aggregates in fairly large, well-defined channels (rivers), it is transmitted downstream by unsteady flow routing techniques of the hydrologic or storage routing variety. Although these routing techniques function adequately in many locations, they have serious shortcomings when the unsteady flows are subjected to backwater effects due to reservoirs, tides, or inflows from large tributaries. Also, when effective hydraulic slopes of the rivers are quite mild, the flow inertial effects ignored in the hydrolgic techniques become important. Also, highly transient flows resulting from dam-breaks which usually greatly exceed the flood-of-record are not treated adequately by the hydrologic routing methods.

To improve the routing capabilities within the NWSRFS, the Hydrologic Research Laboratory in the early 1970's began developing numerical hydrodynamic models suitable for efficient operational use in a wide variety of applications. Two basic models have been developed. The first is a generalized unsteady flow hydrodynamic model known as DWOPER (Dynamic Wave Operational model) suited for river systems. The second is a special unsteady flow model for predicting dam-break floods. The dam-break model, known as DAMBRK, develops the outflow hydrograph from a breached-dam including spillway outflows and routes the flow through the downstream valley using a numerical hydrodynamic model.

The DWOPER model has been applied to many large rivers such as the Mississippi, Ohio, Missouri, Arkansas, Columbia, Cumberland, Tennessee, Susquehanna, and St. John's by the NWS as well as other federal agencies and private consultants. The DAMBRK model is currently being used by the NWS as well as several federal agencies including the Corps of Engineers, Water and Power Resources Service, TVA, and Federal Energy Regulatory Commission to develop flood inundation maps and evacuation plans for major dams within the U.S. Also, many state agencies are using DAMBRK for similar purposes for the myriad of small dams located within each state. Private consultants and Canadian and Central American agencies are likewise using the DAMBRK model.

Burnash, R.J.C., R.L. Ferral, and R.A. McGuire: A generalized streamflow simulation system, Joint Federal-State River Forecast Center, Sacramento, Calif., 204 pp., 1973.

Anderson, E.A.: National Weather Service River Forecast System--snow accumulation and ablation model, NOAA Tech. Memo NWS HYDRO-17, U.S. Dept. Commerce, NOAA, National Weather Service, Silver Spring, Md., 217 pp., 1973.

Both DWOPER and DAMBRK are one-dimensional hydrodynamic models based on an implicit finite difference solution of the conservation form of the St. Venant equations of unsteady flow. The equations consist of the conservation of mass equation, i.e.,

$$\frac{\partial Q}{\partial x} + \frac{\partial (A + A_o)}{\partial t} - q = 0 \tag{1}$$

and the conservation of momentum equation, i.e.,

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\left(\frac{\partial h}{\partial x} + S_f + S_e\right) - q\, v_x + W_f\, B = 0 \tag{2}$$

where:

$$S_f = \frac{n^2 |Q| Q}{2.2\ A^2\ R^{4/3}} \tag{3}$$

$$S_e = \frac{K\ \partial (Q/A)^2}{2g\ \partial x} \tag{4}$$

$$W_f = C_w (V_r \cos w)^2 \tag{5}$$

in which x is distance along the axis of the river, t is time, Q is discharge, A is active cross-sectional area, A_o is inactive (off-channel storage) cross-sectional area, q is lateral inflow (positive) or outflow (negative), g is the gravity acceleration constant, h is water surface elevation, B is wetted top width of cross-section, v_x is velocity of lateral inflow in direction of river-axis (x-direction), S_f is friction slope computed from Manning's equation, n is the Manning n, R is the hydraulic radius approximated by (A/B), S_e is the local loss slope, K is an expansion (negative)--contraction (positive) coefficient, W_f is the wind term, C_w is non-dimensional wind coefficient, V_r is the velocity of the wind relative to the velocity of the river flow, and w is angle between the wind direction and river flow direction. A 4-point weighted, implicit difference approximation is used to transform the non-linear partial differential equations of unsteady flow into non-linear algebraic equations. The 4-point weighted difference approximations are:

$$\frac{\partial K}{\partial t} \simeq \frac{K_i^{j+1} + K_{i+1}^{j+1} - K_i^j - K_{i+1}^j}{2\Delta t} \tag{6}$$

$$\frac{\partial K}{\partial x} \simeq \frac{\Theta}{\Delta x}\left(K_{i+1}^{j+1} - K_i^{j+1}\right) + \frac{(1-\Theta)}{\Delta x}\left(K_{i+1}^j - K_i^j\right) \tag{7}$$

$$K \simeq \frac{\Theta}{2}\left(K_i^{j+1} + K_{i+1}^{j+1}\right) + \frac{(1-\Theta)}{2}\left(K_i^j + K_{i+1}^j\right) \tag{8}$$

where K is a dummy parameter representing any variable in the above differential equations, Θ is a weighting factor varying from 0.5 to 1. To insure unconditional linear numerical stability and provide good accuracy, Θ values nearer to 0.5 are recommended (Fread, 1974). The i subscript denotes a particular

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Fread, D.L.: Numerical properties of implicit four-point finite difference equations of unsteady flow, NOAA Tech. Memo NWS HYDRO-18, U.S. Dept. of cross-section located along the river, and the j superscript denotes a particular time within the solution domain. This scheme was first introduced by Priessmann (1961) and is quite popular among modellers using implicit schemes, e.g., Amein and Fang (1970), Fread (1973), and many others.

The resulting non-linear equations are applied to Δx reaches between river sections having specified cross-sectional properties; the resulting system of algebraic equations are solved by the Newton-Raphson quadratic iterative technique (Amein and Fang, 1970). A special compact quad-diagonal matrix solution technique (Fread, 1971) of the Gauss elimination variety is used to provide optimal efficiency in solving the simultaneous system of linear equations generated within the Newton-Raphson technique. The solution of the finite difference equations provides the water surface elevation and discharge at each specified cross-section. The solutions are obtained when successive iterative values (usually 1-2) of elevation and discharge change less than a specified tolerance value. The Δx reach lengths between cross-sections can be unequal. The solutions are obtained at finite intervals of time as the solution is marched forward in time by Δt steps which can be variable. The time steps can be selected according to accuracy considerations and need not be selected according to the numerical stability constraints associated with explicit finite difference solution techniques.

DWOPER MODEL DESCRIPTION

The DWOPER model contains several features which facilitate its application to a wide variety of unsteady flows occurring in rivers, reservoirs, or estuaries.

Cross-Sections

Cross-sections of irregular as well as regular geometrical shape are acceptable in DWOPER. Each cross-section is read-in as tabular values of channel width and elevation, which together constitute a piece-wise linear relationship. Experience has shown that in almost all instances the cross-section may be sufficiently described with eight or less sets of widths and associated elevations. A low-flow cross-sectional area which can be zero is used to describe the cross-section below the minimum elevation read-in. From this input, the cross-sectional area associated with each of the widths is initially computed within the model. During the solution of the unsteady flow equations, any areas or widths associated with a particular water surface elevation are linearly

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Priessmann, A.: Propagation of translatory waves in channels and rivers, Paper presented at First Congress of French Assoc. for Computation, Grenoble, Sept. 14-16, Proceedings, AFCAL, pp. 433-442, 1961.

Amein M. and C.S. Fang: Implicit flood routing in natural channels, Journ. Hydraulics Div., ASCE, 96, HY12, Dec. pp. 2481-2500, 1970.

Fread, D.L.: Techniques for implicit dynamic routing in rivers with major tributaries, Water Resources Research, 9, No. 4, pp. 918-926, 1973.

Fread, D.L.: Discussion of implicit flood routing in natural channels by M. Amein and C.S. Fang, Journ. Hydraulics Div., ASCE, 97, HY7, July, pp. 1156-1159, 1971.

interpolated from the piece-wise linear relationships of width and elevation read-in or the area-elevation sets initially generated within the model.

Off-Channel Storage

Dead storage areas wherein the flow velocity in the x-direction is considered negligible relative to the velocity in the active area of the cross-section is a feature of DWOPER. Such dead or off-channel storage areas can be used to effectively account for embayments, ravines, or tributaries which connect to the flow channel but do not pass flow and serve only to store the flow. Another effective use of off-channel storage is to model a heavily wooded flood plain which stores a portion of the flood waters passing through the channel. In each of these cases, the use of zero velocity for the portion of the flood waters contained in the dead storage area results in a more realistic simulation of the actual flow than using an average velocity derived from the main flow channel and the dead storage area. The dead storage cross-sectional properties are described similarly to the active cross-sectional areas.

Roughness Coefficients

The Manning n is used to describe the resistance to flow caused by bed forms, bank vegetation and obstructions, bend effects, and eddy losses. The Manning n is defined for each channel reach bounded by gaging stations and is specified as a function of either stage or discharge via a piece-wise linear relation specified as input in tabular form. Linear interpolation is used to obtain n for values of h or Q intermediate to the tabular values. Simulation results are often sensitive to the Manning n. Although in the absence of necessary data (observed stages and discharges), n can be estimated, best results are obtained when n is adjusted to reproduce historical observations of stage and discharge. Such an adjustment process is known as calibration which may be either trial-and-error or an automatic technique described later.

Lateral Inflows

DWOPER incorporates tributary inflows via the lateral inflow term, q, in Eqs. (1-2). These are considered independent of flows occurring in the river to which they are added. They are read-in as a time series of flows with constant or variable time intervals. They may be specified for any Δx reach along the river. Outflows may be simulated by assigning a negative sign to the flow. Linear interpolation is used for flows at times other than the input intervals.

Initial Conditions

Initial conditions consisting of stage (water surface elevation) and discharge at each cross-section must be provided before the unsteady flow equations can be solved. DWOPER provides initial conditions via any of the following: 1) Estimated stage and discharge at each cross-section are read-in; 2) observed stage at each cross-section where a gaging station is located and discharge at the most upstream cross-section of the main stem river and each tributary are read-in; the remaining stages are automatically provided via linear interpolation and the discharges are determined automatically by summation of the flows from upstream to downstream including tributary and/or lateral inflows; 3) computed stages and discharges which have been saved from a previous unsteady flow simulation; and 4) assumed steady flow and a backwater computation to obtain stages upstream of that at the downstream cross-section which is read-in. The equation used for the backwater computation is:

$$(Q^2/A)_{i+1} - (Q^2/A)_i + g/2(A_i+A_{i+1})\left[h_{i+1} - h_i + \frac{n^2\Delta x_i\,(Q_i+Q_{i+1})^2\ (B_i+B_{i+1})^{4/3}}{2.2\ (A_i+A_{i+1})^{10/3}}\right] = 0 \quad (9)$$

in which h_i is the unknown and A_i and B_i are known functions of h_i. Eq. (9) is solved by Newton-Raphson; it usually requires from one to three iterations.

Boundary Conditions

Boundary conditions must be specified in order to obtain solutions to the unsteady flow equations. In fact, in most unsteady flow problems, the unsteady disturbance is introduced into the flow at the boundaries or extremities of the river system. DWOPER can readily accommodate either of the following boundary conditions at the upstream extremities of the river system: 1) known discharge hydrograph, $Q_1^{j+1} - Q(t) = 0$; 2) known stage hydrograph, $h_1^{j+1} - h(t) = 0$. Downstream boundary conditions can include one of the following: 1) known discharge hydrograph, $Q_N^{j+1} - Q(t) = 0$; 2) known stage hydrograph, $h_N^{j+1} - h(t) = 0$; and a known relationship between stage and discharge, i.e., a rating curve. The rating may be single-valued and read-in as tabular (piece-wise linear) values of stage and discharge; linear interpolation is used for intermediate values. The rating may also be a loop rating curve generated internally from cross-section and roughness properties of the downstream boundary section and the instantaneous water surface slope at the previous time step, i.e.,

$$Q_N^{j+1} - \frac{1.486}{n}\left(\frac{A^{5/3}}{B^{2/3}}\right)_N^{j+1} (h_{N-1}^j - h_N^j)\ /\Delta x_{N-1} = 0 \quad (10)$$

where N designates the downstream most cross-section. Hydrographs of stage or discharge may be read-in at constant or specified irregular time intervals.

Enhancement of Computational Algorithm

DWOPER has automatic procedures contained within the finite difference solution algorithm to increase the robust nature of the four-point implicit method. Rapidly rising hydrographs and non-linear properties of the cross-sections due to variations in the vertical and/or along the x-axis may cause computational problems which are manifested by non-convergence in the Newton-Raphson iteration or by erroneously low computed depths at the leading edge of steep-fronted waves. When either of these manifestations are sensed, an automatic procedure consisting of two parts is implemented. The first reduces the current time step (Δt) by a factor of 1/2 and repeats the computations. If the same problem persists, Δt is again halved and the computations repeated. This continues until a successful solution is obtained or the time step has been reduced to 1/16 of the original size. If a successful solution is obtained, the computational process proceeds to the next time level using the original Δt. If the solution using $\Delta t/16$ is unsuccessful, the Θ weighting factor is increased by 0.1 and a time step of $\Delta t/2$ is used. Upon achieving a successful solution, Θ and the time step are restored to their original values. Unsuccessful solutions are treated by increasing Θ and repeating the computation until $\Theta = 1.0$ whereupon the automatic procedure terminates and the solution with $\Theta = 1.$ and $\Delta t/2$ is used to advance the solution forward in time now using the original Θ and Δt values. Often computational problems can be overcome via one or two reductions in the time step.

Lock and Dam Internal Boundary Condition

A river system may include small dams with gates to pass the river flow in such a way as to maintain desired water surface elevations on the upstream side of the dam. A lock is provided for navigation of river craft past the dam. DWOPER can accommodate any number of lock and dam (L&D) installations within the river system being simulated. An internal boundary condition is used as opposed to separating the river system into discrete portions and specifying external boundary conditions applicable to the L&D. The internal boundary allows the simultaneous simulation of the entire river system including any L&D. If the pool elevation is controlled only by the gate operation, Eq. (1) is replaced by:

$$Q_i^{j+1} - Q_{i+1}^{j+1} = 0 \tag{11}$$

and the conservation of momentum equation, Eq. (2), is replaced by:

$$h_i^{j+1} - h_t = 0 \tag{12}$$

where h_t is the target pool elevation which the dam operator attempts to maintain via operation of the gates. The target pool elevation may be a constant value, or it may be specified as a function of time and read-in as a time series. When the simulated tailwater elevation exceeds a specified critical tailwater elevation, the flow is computed as governed by Eqs. (1) and (2).

Dendritic River Systems

An efficient solution technique for dendritic (tree-type) river systems is utilized in DWOPER. This technique solves during a time step the unsteady flow equations first for the main stem, and then for each tributary of the river system. The tributary flow at the confluence with the main-stem river is treated as lateral flow q which is first estimated when solving the equations for the main stem. The tributary flow depends on its upstream boundary condition, lateral inflows along its reach, and the water surface elevation at the confluence (downstream boundary for the tributary) which is obtained during the simulation of the main stem. Due to the interdependence of the flows in the main stem and its tributaries, the following iterative or relaxation algorithm is used:

requires about two times that of the quad-diagonal and only 0.0037 that of the full Gaussian. Of course, the multiple-channel formulation feature of DWOPER can be used for purely dendritic river systems having any order of tributaries.

Weir-Flow Bifurcations

In DWOPER, any number of Δx reaches along a channel may bypass flow to another channel which may or may not connect back into the former channel at some point downstream from the bifurcation. The flow in the bypass channel which may affect the weir flow is accounted for by a submergence correction factor (K_s). In fact, depending on the relative elevations of the water surface in the bypass channel (h_{bc}) and the main channel (h), the flow can reverse and flow back into the main channel. The crest elevation (h_c) of the overbank section which acts as the weir-flow bypass is specified. Each section has a discharge coefficient (C) which may be estimated or obtained through trial-and-error calibration. The location along the channel where the bifurcation(s) occur, the average crest elevation of each such Δx reach, and the discharge coefficient are read-in as input data. The weir-flow equation is:

$$q = C\,K_s\,(h - h_c)^{3/2} \tag{20}$$

where:

$$K_s = 1.0 \qquad \gamma \leq 0.67 \tag{21}$$

$$K_s = 1.0 - 27.8\,(\gamma - 0.67)^3 \qquad \gamma > 0.67 \tag{22}$$

$$\gamma = (h_{bc} - h_c)/(h - h_c) \tag{23}$$

The weir-flow bifurcation can be used to simulate levee overtopping and time-dependent levee crevasses. The overtopping and/or breach flow is then routed through the flood plain which is considered to be a tributary. The tributary may be hydraulically connected to the main river via a natural confluence or a flap-gated gravity drainage pipe. Also, the tributary may not be connected hydraulically with the main river; in this case the flow merely ponds within the flood plain. Levees may be located on both sides of the flooding river.

Automatic Calibration

A critical task in the application of one-dimensional hydrodynamic models in natural rivers in dendritic systems is the determination of the Manning n which often varies with discharge or stage, and with distance along the river. DWOPER has an option to automatically determine the optimum Manning n which will minimize the difference between computed and observed values via a highly efficient optimization technique (Fread and Smith, 1978). The Manning n may be constant or have a piece-wise linear variation with discharge (or stage) for each reach of the river bounded by gaging stations. The optimization technique is based on a scheme of decomposing complex river systems of dendritic configuration. Computational requirements are less than twice that required for a normal simulation run.

In automatic calibration, optimum Manning n values are sequentially

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Fread, D.L. and G.F. Smith: Calibration technique for one-dimensional unsteady flow models. Journ. Hydraul. Div. ASCE 104 HY7 July pp 1027-1043 1978

$$q^* = \alpha\, q + (1-\alpha)\, q^{**} \tag{13}$$

in which α is a weighting factor ($0 < \alpha \leq 1$), q is the computed tributary flow at the confluence, q^{**} is the previous estimate of q, and q^* is the new estimate of q. Convergence is attained when q is sufficiently close to q^{**}. Usually, one or two iterations is sufficient; however, the α weighting factor has an important influence on the algorithm's efficiency. Optimal values of α can reduce the iterations by as much as 1/2. A priori selection of α is difficult since α varies with each river system. Good first approximations for α are in the range, $0.6 \leq \alpha \leq 0.8$. DWOPER can accommodate any number of 1st order tributaries. Systems with 2^{nd} order tributaries can sometimes be accommodated by reordering the system, i.e., selecting another branch of the system as the main stem.

Multiple-Channel System

When the river system consists of bifurcations due to islands, man-made bypasses, etc., such that the river system is not simply dendritic, an alternative computational formulation is used; it is based on three internal boundary equations at each junction, i.e.,

$$Q_1 + Q_2 - Q_3 + \Delta s/\Delta t = 0 \tag{14}$$

$$Q_1^{\,2}/(2gA_1^{\,2}) + h_1 - Q_2^{\,2}/(2gA_2^{\,2}) - h_2 = 0 \tag{15}$$

$$Q_2^{\,2}/(2gA_2^{\,2}) + h_2 - Q_3^{\,2}/(2gA_3^{\,2}) - h_3 = 0 \tag{16}$$

in which subscripts denote the three river branches entering and exiting a junction, and (Δs) is the change in the junction storage. It is not possible to maintain a diagonally banded matrix for the coefficients introduced via Eqs. (14-16); however, by using a unique numbering scheme for the cross-sections within the river system and by properly introducing Eqs. (14-16) in the composition of the coefficient matrix, the number and consequence of off-diagonal elements can be minimized. Then, using a special matrix solution technique that operates only on non-zero off-diagonal elements, an optimally efficient solution of the matrix can be achieved. This method of simulating multiple channels maintains a nonlinear formulation of the entire system, thus retaining the Newton-Raphson iterative equation solver, and yet performs the computations quite efficiently. Expressions for total number of operations (T) in solving the matrix are shown below for the quad-diagonal algorithm used for only dendritic systems or a single channel in DWOPER, the multiple-channel algorithm, and for comparison a full Gaussian elimination algorithm:

$$T_1 \simeq 38\,N \;\ldots\ldots\ldots\ldots\ldots\ldots\; \text{(quad-diagonal algorithm)} \tag{17}$$

$$T_2 \simeq 66\,N + (26\,N + 25)\,J_n \;\ldots\ldots\; \text{(multi-channel algorithm)} \tag{18}$$

$$T_3 \simeq \frac{16}{3} N^3 + 4N^2 \;\ldots\ldots\ldots\ldots\; \text{(full Gaussian algorithm)} \tag{19}$$

in which N is number of cross-sections, J_n is number of junctions. For example, if N = 100 and J_n = 5, T_1 = 3800 x 2.5 = 9,500, T_2 = 19,725, and T_3 = 5,373,333;

requires about two times that of the quad-diagonal and only 0.0037 that of the full Gaussian. Of course, the multiple-channel formulation feature of DWOPER can be used for purely dendritic river systems having any order of tributaries.

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Fread, D.L. and G.F. Smith: Calibration technique for one-dimensional unsteady flow models. Journ. Hydraul. Div. ASCE 104 HY7 July pp 1027-1043 1978 determined for each reach bounded by gaging stations, commencing with the most upstream reach, and progressing reach by reach in the downstream direction. Tributaries are calibrated before the main stem river and their flows are added to the main stem as lateral inflows. Discharge is input at the upstream boundary of each river, while observed stages at the downstream gaging station of each reach is used as the downstream boundary condition. Computed stages at the upstream boundary are tested against observed stages at that point. Statistics of bias (Φ_j) and root-mean-square (RMS) error are computed for several (j) ranges of discharge or stage so that the Manning n can be calibrated as a function of discharge or stage. For each range of discharge, an improved estimate of the optimum Manning n (n_j^{k+1}) is obtained via a modified Newton-Raphson iterative algorithm, i.e.:

$$n_j^{k+1} = n_j^k - \frac{\Phi_j^k \, (n_j^k - n_j^{k-1})}{\Phi_j^k - \Phi_j^{k-1}} \qquad k \geq 2; \; j = 1,2,\ldots J \tag{24}$$

in which the k superscript denotes the number of iterations and Φ_j is the bias for the j^{th} range. Eq. (24) can be applied only for the second and successive iterations; therefore, the first iteration is made using the following algorithm:

$$n_j^{k+1} = n_j^k \, (1.0 - 0.01 \; \Phi_j^k / |\Phi_j^k|) \qquad k = 1; \; j = 1,2,\ldots J \tag{25}$$

in which a small percentage change in n is made in the correct direction as determined by the term $(-\Phi_j^k/|\Phi_j^k|)$. The convergence properties of Eq. (24) are quadratic with convergence usually obtained within 3 - 5 iterations. Improved n values obtained via Eq. (24) are used and the cycle repeated until a minimum RMS error for the reach is found. Then, the discharges computed at the downstream boundary using the optimum Manning n are stored internally and then input as the upstream boundary condition for the next downstream reach.

DAMBRK MODEL DESCRIPTION

The DAMBRK model (Fread, 1977) synthesizes the outflow hydrograph due to a dam failure and/or spillway flows. The earthen or concrete dam is assumed to fail by either overtopping, piping, or collapse within a specified interval of time. The time-dependent geometry of the breach is specified by simple parameters to describe a complete or partial breach. The effects of reservoir inflows, reservoir storage, and spillway outflows on the broad-crested weir flow through the breach are accounted for via selection of one of two reservoir routing techniques, i.e., storage routing (level pool) or dynamic routing (St. Venant equations). The effect of tailwater on the outflow is modelled with a submergence correction factor. The outflow hydrograph is routed through the downstream valley via the 4-point implicit difference solution of the conservation form of the St. Venant equations. (This is the same as in the DWOPER model.) Provisions

Fread, D.L.,: The development and testing of a dam-break flood forecasting model, Proc., Dam-Break Flood Modeling Workshop, U.S. Water Res. Council, Wash., D.C., pp. 164-197, 1977.

are included in the model for routing supercritical or subcritical flows and accounting for tributary inflows. The effects of downstream obstructions such as road embankments, bridges, and/or other dams are modelled as internal boundary conditions. The model can create additional cross-sections via linear interpolation between the specified cross-sections; this option is an important convenience when modelling steep-fronted waves which require frequent computational points along the river. Composite cross-sections with optional inactive flow (storage) areas can be used with the St. Venant equations, or a modified version of the St. Venant equations can be used to better treat substantial flood-plain flows and the effects of a meandering river channel.

Breach Description

The DAMBRK model allows the user to input the failure time interval (τ) and the terminal size and shape of the breach. The shape is specified by (z), the side slope of the breach, i.e., 1 vertical: z horizontal. Rectangular, triangular, or trapezoidal shapes may be specified. The final breach size is controlled by z and the parameter (BB) which is the terminal width of the bottom of the breach at elevation h_{bm}. The model assumes the breach bottom width starts at a point and enlarges to BB at a linear rate over the failure time interval (τ).

During the simulation of a dam failure, the actual breach formation commences when the water surface elevation (h) within the reservoir exceeds a specified value, h_f. This feature permits the simulation of overtopping a dam in which the breach does not form until a sufficient amount of water flows over the crest of the dam. A piping failure may be simulated when h_f is a specified elevation less than the height of the dam, h_d.

Reservoir Outflow Hydrograph

The total reservoir outflow (Q) consists of broad-crested weir flow through the breach (Q_b) and flow through any spillway outlets (Q_s),

where: $$Q_b = K_s c_v [3.1\ BB\ t_b/\tau (h-h_b)^{1.5} + 2.45\ z(h-h_b)^{2.5}] \tag{26}$$

$$Q_s = K_s c_s (h-h_g)^{1.5} + c_g (h-h_g)^{0.5} + c_d (h-h_d)^{1.5} + Q_t \tag{27}$$

$$c_v = 1.0 + 0.023\ Q^2/[B_d^{\,2} h^2 (h-h_b)] \tag{28}$$

in which h_b is the breach bottom elevation evaluated as: $h_b = h_d - (h_d - h_{bm})\ t_b/\Upsilon$, t_b is the time after the breach starts forming, c_v is the correction for velocity of approach, B_d is the width of the reservoir at the dam, K_s is the submergence correction for tailwater effects on the breach and spillway outflow, and h_t is the tailwater elevation (water surface elevation immediately downstream of dam), c_s is the uncontrolled spillway discharge coefficient, h_s is the uncontrolled spillway crest elevation, c_g is the gated spillway discharge coefficient, h_g is the center-line elevation of the gated spillway, c_d is the discharge coefficient for flow over the crest of the dam, and Q_t is a constant (head independent) outflow or leakage.

DAMBRK can use storage routing to compute the reservoir outflow, i.e.,

$$I - Q = dS/dt \tag{29}$$

in which I is the reservoir inflow, Q is the total reservoir outflow, and dS/dt is the time rate of change of reservoir storage volume. Expressing Eq. (29) in centered finite difference form where a prime (') superscript denotes values at the time (t-Δt) and approximating S in terms of A_s (the reservoir surface area) result in the following expression:

$$(A_s + A'_s)\ (h-h')/\Delta t + Q + Q' - I - I' = 0 \tag{30}$$

Since Q and A_s are functions of h and all other terms are known, Eq. (30) can be solved for the unknown h using Newton-Raphson iteration. Once h is obtained, Eqs. (26) and (27) can be used to obtain the total outflow (Q) at time (t).

Modified St. Venant Equations for Flood-Plain Flows

The St. Venant equations are modified (Fread, 1976) as follows:

$$\frac{\partial(K_c Q)}{\partial x_c} + \frac{\partial(K_\ell Q)}{\partial x_\ell} + \frac{\partial(K_r Q)}{\partial x_r} + \frac{\partial A}{\partial t} - q = 0 \tag{31}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial(K_c^2 Q^2/A_c)}{\partial x_c} + \frac{\partial(K_\ell^2 Q^2/A_\ell)}{\partial x_\ell} + \frac{\partial(K_r Q^2/A_r)}{\partial x_r} + gA_c\left(\frac{\partial h}{\partial x_c} + S_{fc} + S_e\right)$$
$$+ gA_\ell\left(\frac{\partial h}{\partial x_\ell} + S_{f\ell}\right) + gA_r\left(\frac{\partial h}{\partial x_r} + S_{fr}\right) = 0 \tag{32}$$

The parameters (K_c, K_ℓ, K_r) proportion the total flow (Q) into the channel, left flood plain, and right flood plain, respectively. These are defined as follows:

$$K_c = 1/(1 + k_\ell + k_r) \tag{33}$$

$$K_\ell = k_\ell/(1 + k_\ell + k_r) \tag{34}$$

$$K_r = k_r/(1 + k_\ell + k_r) \tag{35}$$

and,

$$k_\ell = \frac{Q_\ell}{Q_c} = \frac{n_c}{n_\ell}\frac{A_\ell}{A_c}\left(\frac{R_\ell}{R_c}\right)^{2/3}\left(\frac{\Delta x_c}{\Delta x_\ell}\right)^{1/2} \tag{36}$$

$$k_r = \frac{Q_r}{Q_c} = \frac{n_c}{n_r}\frac{A_r}{A_c}\left(\frac{R_r}{R_c}\right)^{2/3}\left(\frac{\Delta x_c}{\Delta x_r}\right)^{1/2} \tag{37}$$

Eqs. (36) and (37) represent the ratio of flow in the channel section to flow in the left and right flood-plain sections, where the flows are expressed in terms of the Manning equation with the energy slope approximated by the water surface slope (Δh/Δx). The friction slope terms in Eq. (32) are similar to Eq. (3).

Multiple Dams and Bridges

The DAMBRK model can simulate the progression of a dam-break wave through a downstream valley containing a reservoir created by another downstream dam, which itself may fail due to being sufficiently overtopped by the wave emanating from

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Fread, D.L.: Flood routing in meandering rivers with flood plains, Proceedings, Rivers '76, 3rd Ann. Symp of Waterways, Harbors & Coastal Eng. Div., ASCE, Vol. I, Aug., pp. 16-35, 1976.

the failure of the upstream dam. In fact, an unlimited number of reservoirs located sequentially along the valley can be simulated. In this method a dam is treated as an internal boundary condition where flow through a short Δx reach containing the dam is governed by Eq. (11) and the following:

$$Q_i = Q_b + Q_s \tag{38}$$

in which Q_b and Q_s are breach and spillway flow described by Eqs. (26-27).

Highway/railway bridges and their earthen embankments located downstream of a dam may also be treated as internal boundary conditions. Eqs. (11) and (38) are used at each bridge; the term Q_s in Eq. (38) is defined as follows:

$$Q_s = C_b\sqrt{2g}\, A_{i+1}(h_i - h_{i+1})^{1/2} + C_d k_s (h - h_c)^{3/2} \tag{39}$$

in which C_b is a coefficient of bridge flow, C_d is the coefficient of flow over the road embankment, h_c is the crest elevation of the embankment, and k_s is a submergence correction factor similar to K_s as defined in Eqs. (21) and (22).

Supercritical Flow

Downstream valley slopes greater than about 0.01 usually result in the flow being supercritical. Therein, unlike subcritical flow the downstream boundary is not required since flow disturbances cannot travel upstream. However, in addition to the reservoir outflow, another upstream boundary condition similar to Eq. (10) is used in DAMBRK.

Landslide-Generated Waves

Waves within reservoirs generated by landslides can be simulated in DAMBRK. The volume of the landslide mass, its porosity, and time interval over which the landslide occurs are input to the model. The landslide mass is deposited within the reservoir in layers during small computational time steps, and simultaneously the original dimensions of the reservoir are reduced accordingly. The time rate of reduction in the reservoir cross-sectional area creates the wave during the solution of the unsteady flow equations.

CONCLUDING REMARKS

The DWOPER model has generally reproduced observed stage hydrographs within 1-2 percent, and the DAMBRK model has reproduced observed flood crests within 2-15 percent. The computational efficiency of both models is most satisfactory; over a wide range of applications typical CPU requirements are less than 1 minute.

Current river mechanics research in the Hydrologic Laboratory is the expansion of the applicability of both models. The DWOPER model is being extended to include sediment transport effects on hydraulic roughness and cross-section geometry, ice jam and break-up effects on unsteady flows, flow proportioning into channel and flood-plain for meandering rivers, bank storage and overbank infiltration losses. Updating algorithms such as the Kalman filter technique are being investigated for enhancing the use of DWOPER in real-time flood forecasting. The DAMBRK routing algorithm is being improved for applications to highly irregular cross-sections and mixed subcritical-supercritical flow.